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Fractal and Mathematical Inductive Diffraction Patterns in SLM

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Abstract. We simulate diffraction patterns (fringes and square apertures) from a generating base and the necessary conditions to create an arbitrary fringes number and apertures. We study the mathematical properties of such construction and compared them with the properties of a fractal. Finally, from the fractal consideration and apertures size, we obtain that the Fraunhofer diffraction analysis for the LCD as a diffractive element is a mathematical inductible pattern.

Keywords: Diffraction patterns, liquid crystal spatial light modulator, fractal, Fraunhofer diffraction analysis, mathematical inductible pattern.

1 Introduction

In this work, we build an algorithm to simulate and LCSLM (Liquid Crystal Spatial Light Modulator-SLM, which we will refer to as LCD) without considering its chemistry and all the proper polarization elements corresponding to the arrays involving SLM [10, 16, 7, 1]. We idealized the LCD resembling a fractal [5, 17], from a base constituted by a square of transmission equal to 1 surrounded by 0-transmission squares. Subsequently, we validated the LCD from its diffractive features [1, 13]. We propose a mathematically inductible high-order diffractive process [8] to validate the $N \times N$ resolution of the simulated LCD.

2 Methodology

We do not consider the polarization effects for the sake of the LCD design as a simulated diffractive element since numerically, the wavefront can be built and visualized straightforwardly. We bear this in mind since it is a common practice to place the polarizer before and after the LCSLM, as in [1, 13].

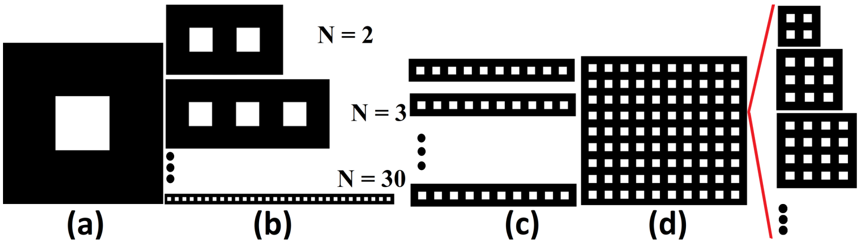

Fig. 1. We show the design and construction of our LCD, simulated from the base (a) with nine zero-valued squares surrounding a square with a value equal to 1. In (b), we show the base, generating pixels rows from square 1 by replicating it up to the desired number of pixels in a row. In (c), we show the replication of the row in (b) to generate similar rows coupling in (d), building the simulated LCD, followed by each pixel, previously described and resembling a fractal [5, 17].

On the other hand, we recall that fractals are generated from a base that can be replicated (by multiplying, summing, or joining it) in specific points and as many times as desired given the fractal, and due to the definition of the fractal symmetry and auto-similarity, which are essential fractal features [5, 17].

We show the LCD diffractive base in Fig. 1 (a), constituted by nine squares, eight black ones (opaque or with null transmission and total absorption without reflection, with zero intensity value), surrounding a white square (completely transmissible and non-absorbent, with an intensity equal to 1) resembling a window or an ideal pass-band filter (square-shaped window) [12].

We consider the squares in the base as pixels or matrix entries, as a simple conception to assign proper values simplifying the numerical simulation (completely performed in MATLAB ®in this case). The latter yields a minimum entry or pixel value equal to 1 (considering a square matrix), avoiding problems with the sampling Whittaker-Nyquist-Kotelnikov-Shannon theorem (widely-known as Nyquist-Shannon sampling theorem or Nyquist theorem) [14, 2].

In order to generate all the LCD pixels, we consider the base and integrate more elements to it, with the same features on the right side, up to a $1\times N$-pixels size [according to the desired pixels number N, see Fig. 1 (b)]. Subsequently, we build N replicas of the $1\times N$ row, with the following one placed right before the previous one, and each pixel abiding by the base and the square order 0 and 1 [see Fig. 1 (c)]. Hence, we obtain our $N\times N$-pixels LCD [see Fig. 1 (d)], built from the base [Fig. 1 (a)], resembling a fractal [5, 17].

3 Inductive Diffraction Patterns

We consider the LCD in Fig. 1 (d) as a diffractive object and with a fractal base (only the base) to analyze it by means of far field diffraction theory or Fraunhofer [11, 4, 6]. Hence, we validate the diffraction patterns simulating LCDs with different sizes 1×1 [Fig. 1 (a)], 2×2, 3×3 and 4×4 [Fig. 2 (a)-(c)]. Thus, we consider the diffractive field $U(x,y)$, from the following expression:

$$U\left(x,y\right)=\frac{e^{ikz}}{ikz}e^{i\frac{k}{2z}\left(x^2+y^2\right)}\iint_{-\infty}^{\infty}\widetilde{U_0}\left(\xi,\eta\right)e^{-i\frac{2\pi}{z\lambda}\left(x\xi+y\eta\right)}d\xi d\xi, \tag{1}$$

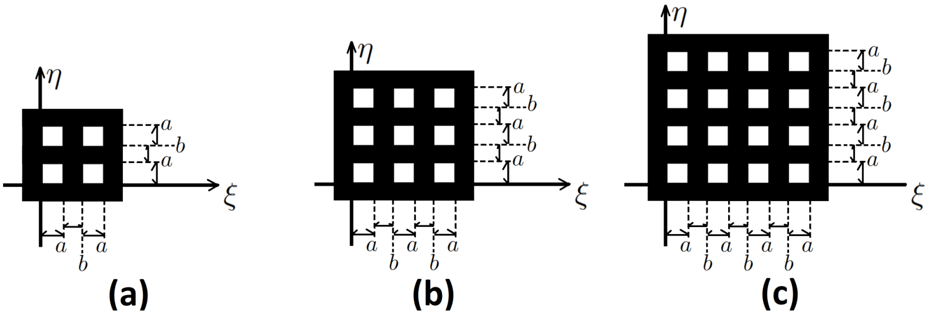

Fig. 2. We show the reference system origin position used to compute the diffractive field and the intensity for the simulated LCD of $N \times N$ pixels with $N = 1, 2, 3, ..., n$ in (a)-(c) and Fig. 1 (d), respectively.

where $\widetilde{U_0}(\xi, \eta)$ represent the diffractive object, with $\lambda = 632$ nm being the wavelength [11, 4, 6]. Define $A = \frac{e^{ikz}}{ikz} e^{i\frac{k}{2z}(x^2+y^2)}$ and $S_{a,b}(x) = e^{-i2\frac{\pi}{z\lambda}x(a+b)}$. By neglecting certain results in the diffraction area, such as $\text{rect}(z/a)$, and with $a = 6\,\mu\text{m}$ on the propagation axis z, we derive the intensities of the diffractive fields as $I(x, y) = \|U(x, y)\|^2$, from the arrays depicted in Fig. 1 (a), Figs. 2 (a)-(c), and Fig. 1 (d). The expressions for these intensities are as follows:

$$I(x, y) = I_{1x1} = \frac{a^4\lambda^2}{4\pi^2 z^2} \text{Sinc}^2\left(a\frac{\pi}{z\lambda}x\right) \text{Sinc}^2\left(a\frac{\pi}{z\lambda}y\right), \quad (2)$$

$$I_{2x2} = \frac{a^4\lambda^2}{4\pi^2 z^2} \text{Sinc}^2\left(a\frac{\pi}{z\lambda}x\right) \text{Sinc}^2\left(a\frac{\pi}{z\lambda}y\right) \left\|\sum_{i=0}^{1} \left[S_{a,b}(x)\right]^i\right\|^2 \left\|\sum_{i=0}^{1} \left[S_{a,b}(y)\right]^i\right\|^2, \quad (3)$$

$$I_{3x3} = \frac{a^4\lambda^2}{4\pi^2 z^2} \text{Sinc}^2\left(a\frac{\pi}{z\lambda}x\right) \text{Sinc}^2\left(a\frac{\pi}{z\lambda}y\right) \left\|\sum_{i=0}^{2} \left[S_{a,b}(x)\right]^i\right\|^2 \left\|\sum_{i=0}^{2} \left[S_{a,b}(y)\right]^i\right\|^2, \quad (4)$$

$$I_{4x4} = \frac{a^4\lambda^2}{4\pi^2 z^2} \text{Sinc}^2\left(a\frac{\pi}{z\lambda}x\right) \text{Sinc}^2\left(a\frac{\pi}{z\lambda}y\right) \left\|\sum_{i=0}^{3} \left[S_{a,b}(x)\right]^i\right\|^2 \left\|\sum_{i=0}^{3} \left[S_{a,b}(y)\right]^i\right\|^2, \quad (5)$$

$$I_{NxN}(x, y) = \frac{a^4\lambda^2}{4\pi^2 z^2} \text{Sinc}^2\left(a\frac{\pi}{z\lambda}x\right) \text{Sinc}^2\left(a\frac{\pi}{z\lambda}y\right) \left\|\sum_{i=0}^{N-1} \left[S_{a,b}(x)\right]^i\right\|^2 \left\|\sum_{i=0}^{N-1} \left[S_{a,b}(y)\right]^i\right\|^2. \quad (6)$$

Hence, from Eq. 6, we can show that the diffractive field intensity $I_{N\times N}$ is inductible, and we can extend the result for an $n \times n$-LCD with n an arbitrary integer, resembling a fractal. The latter is based on the Geometric Series [15], [3], [9], which is a converging series, considering the diffraction principles of the parameter A. Such a series is described by:

$$T^n = \sum_{i=0}^{n} z^i = z^0 + z^1 + z^2 + z^3 + ... + z^n = \sum_{i=0}^{n} z^i = \frac{1 - z^{n+1}}{1 - z}. \quad (7)$$

It is possible to generalize the diffractive field expression in terms of:

$$U_{N\times N}(x,y) = a^2 A A_a(x) A_a(y) \mathrm{Sinc}\left(a\frac{\pi x}{z\lambda}\right)\mathrm{Sinc}\left(a\frac{\pi y}{z\lambda}\right) \times \left\{\frac{1-\exp\left(-iNx\left(\frac{2\pi(a+b)}{z\lambda}\right)\right)}{1-\exp\left(-ix\left(\frac{2\pi(a+b)}{z\lambda}\right)\right)}\right\} \times \left\{\frac{1-\exp\left(-iNy\left(\frac{2\pi(a+b)}{z\lambda}\right)\right)}{1-\exp\left(-iy\left(\frac{2\pi(a+b)}{z\lambda}\right)\right)}\right\}, \tag{8}$$

$$I_{N\times N}(x,y) = \frac{a^4\lambda^2}{4\pi^2 z^2}\mathrm{Sinc}^2\left(a\frac{\pi x}{z\lambda}\right)\mathrm{Sinc}^2\left(a\frac{\pi y}{z\lambda}\right) \times \left\|\frac{1-\exp\left(-iNx\left(\frac{2\pi(a+b)}{z\lambda}\right)\right)}{1-\exp\left(-ix\left(\frac{2\pi(a+b)}{z\lambda}\right)\right)}\right\|^2 \times \left\|\frac{1-\exp\left(-iNy\left(\frac{2\pi(a+b)}{z\lambda}\right)\right)}{1-\exp\left(-iy\left(\frac{2\pi(a+b)}{z\lambda}\right)\right)}\right\|^2. \tag{9}$$

4 Conclusions

We have shown that it is possible to obtain a diffractive field resembling a fractal by considering a diffractive element with typical features of LCSLM or simply and LCD in an inductible diffraction set with its intensity reflecting the properties inherited from the diffractive construction element. In this context, it is not possible to stress the auto similarity of fractal dimension arguments. However, it is possible to analyze the diffractive field intensity, mathematically inductible, if there exists a diffractive element, meeting the periodicity conditions and features of an LCD.

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